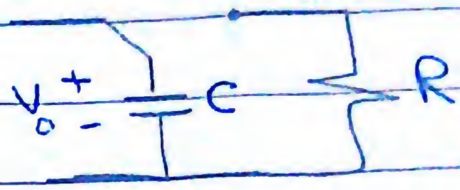
Date: Subject: Lec 4

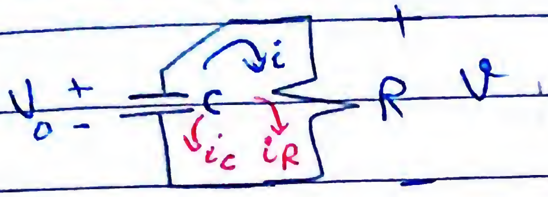
Natural Response of RC Circuits

initial Condition ⇐

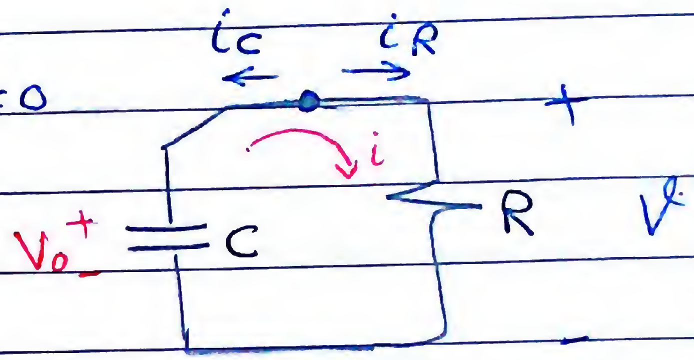
قد يكون موجود أو غير موجود ⇐



response بدأ ← $t=0$



$i_R + i_C = 0$ التيارين خارجين طبقاً لقانون كيرشوف



$i_C + i_R = 0$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\boxed{\frac{dv}{dt} + \frac{1}{CR}v = 0}$$

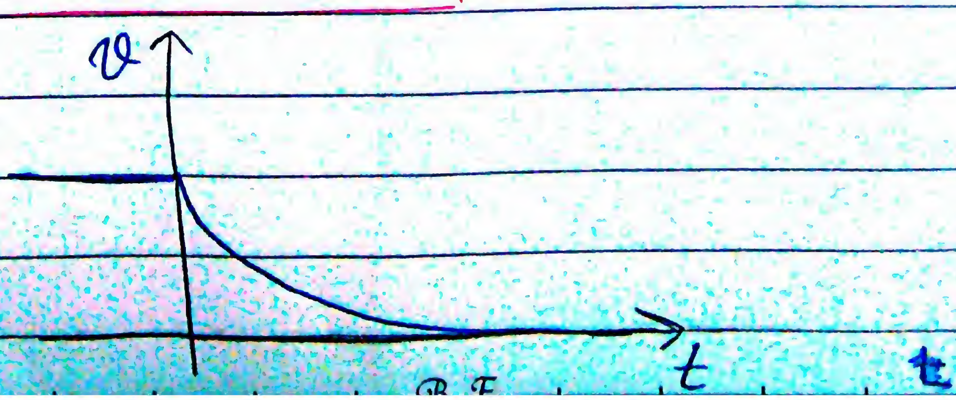
$v(0) = V_0$ at $t=0$

$$v(t) = K e^{-t/\tau}$$

$\tau = CR$

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$

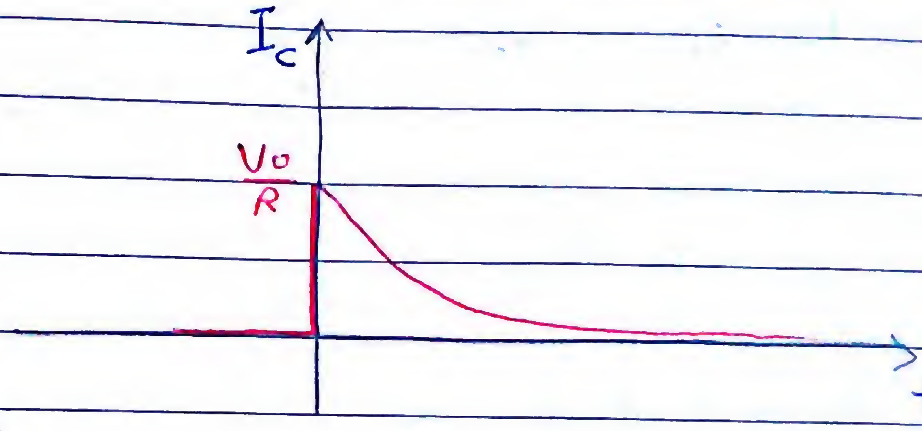
$t \geq 0$



Date: Subject:

$$i = \frac{v(t)}{R} \quad \text{or} \quad i = C\frac{dv}{dt}$$

$$i(t) = \frac{V_0}{R} e^{-t/\tau} \qquad \boxed{t > 0^+}$$

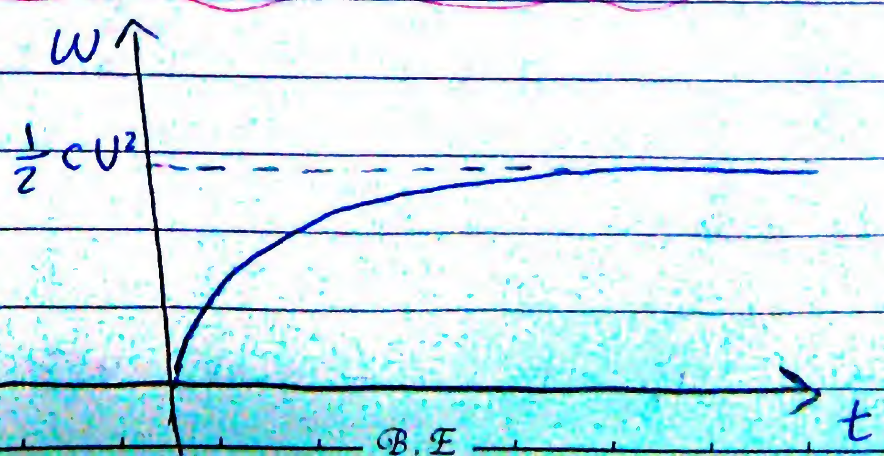


$$P(t) = vi$$

$$P(t) = \frac{V_0^2}{R} e^{-2t/\tau} \qquad \boxed{t > 0^+}$$

$$w(t) = \int_0^t vi\,dt$$

$$w(t) = \frac{1}{2} C V_0^2 \left[1 - e^{-2t/\tau}\right]$$



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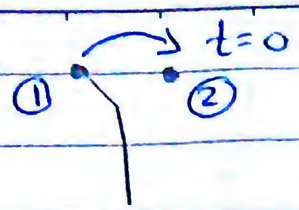
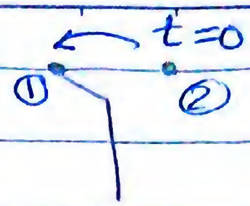
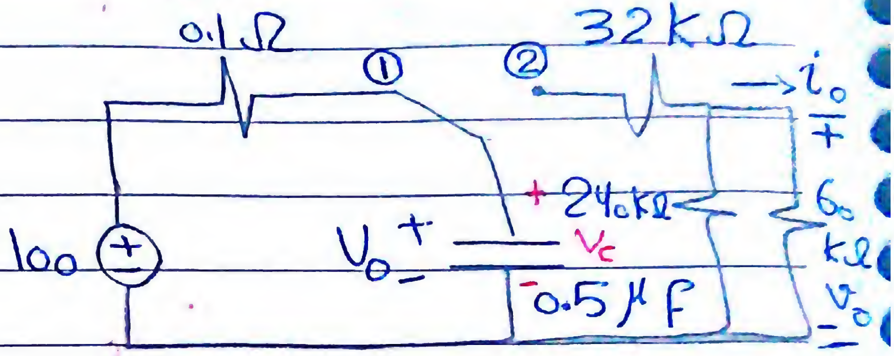




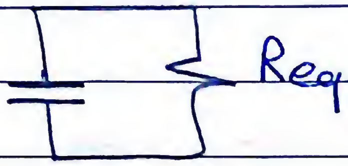
① steady state

المكثف → O.C

$V_o = 100\ V$



②



$R_{eq} = 80\,k\Omega$

$\tau = 80k\Omega * 0.5\,\mu F = 0.04\,s$

$V_c = 100\,e^{-25t} \qquad t \geq 0$

← لان الجهد في المكثف لا يتغير لحظيا

وهو معروف عند $t=0$ & على عكس التيار

$$v_o = \frac{48}{80} * 100\,e^{-25t} \qquad t > 0^+$$

← غير معروف على المقاومة

V_o قبل $t=0$ ⇐ صفر

مسموح للجهد يتغير لحظياً على المقاومة

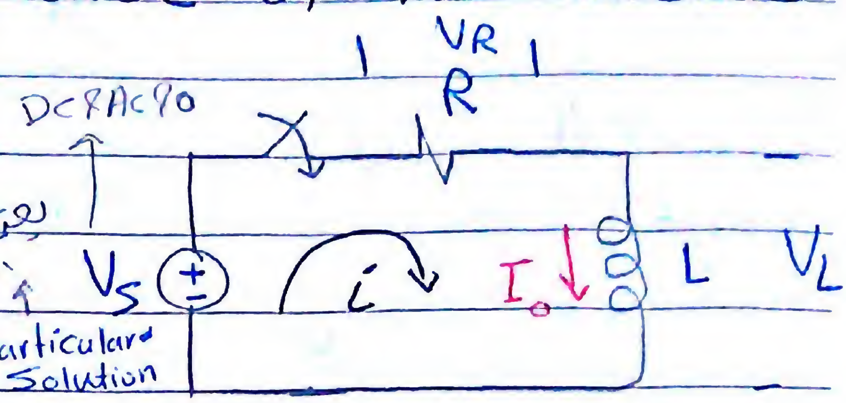
$$i_o = \frac{v_o}{60} = 0.001\,e^{-25t} \qquad t > 0^+$$

$$P_{60k\Omega} = i_o^2 * 60\,k\Omega = \frac{v_o^2}{60\,k\Omega}$$

$$W_{60k\Omega} = \int_0^\infty P_{60}\,dt = 0.012\ WS$$

Date: Subject:

Step Response of RL Circuits "Series"



$$L\frac{di}{dt} + Ri = V_s$$ يعتبر

نعرف (بيتم)

$$i(t) = i_c + i_p \rightarrow \text{Particular Solution}$$

$i_c \rightarrow 0$

Complementary Solution $= i_t + i_{ss} \rightarrow$ Steady State

$\quad \hookrightarrow$ transiant

$$i(t) = Ke^{-t/\tau} + i_{ss}$$

$$i(t) = K e^{-t/\tau} + \frac{V_s}{R}$$

$$i(0) = I_0$$

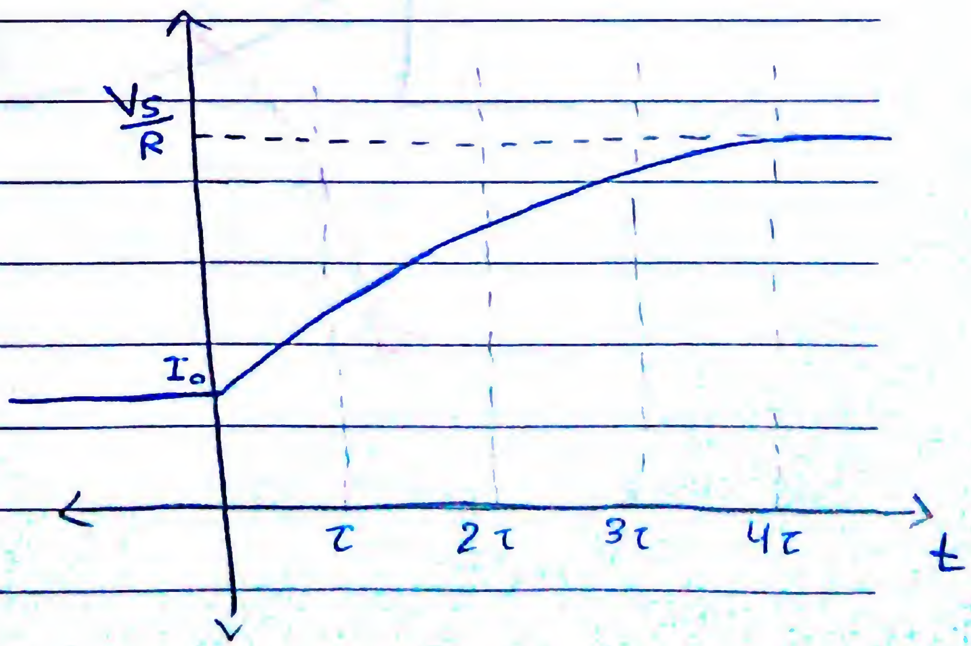
$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-t/\tau}} \qquad \boxed{t \geq 0}$$

لو $I_0 = \frac{V_s}{R}$

ال transiant = 0

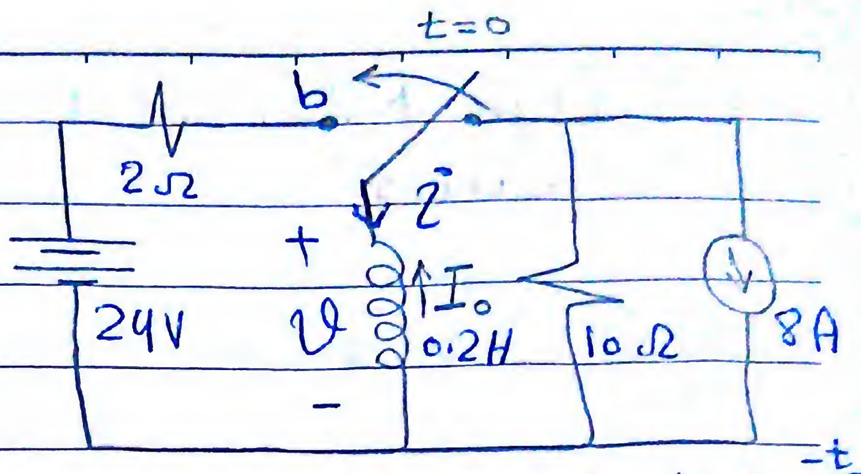
فهيبدأ ال

steady state ع طول



Date: Subject:

Ex:



$I_0 = -8\,A = i(0)$

$i(\infty) = 12\ A$

$\tau = 0.1\,s$

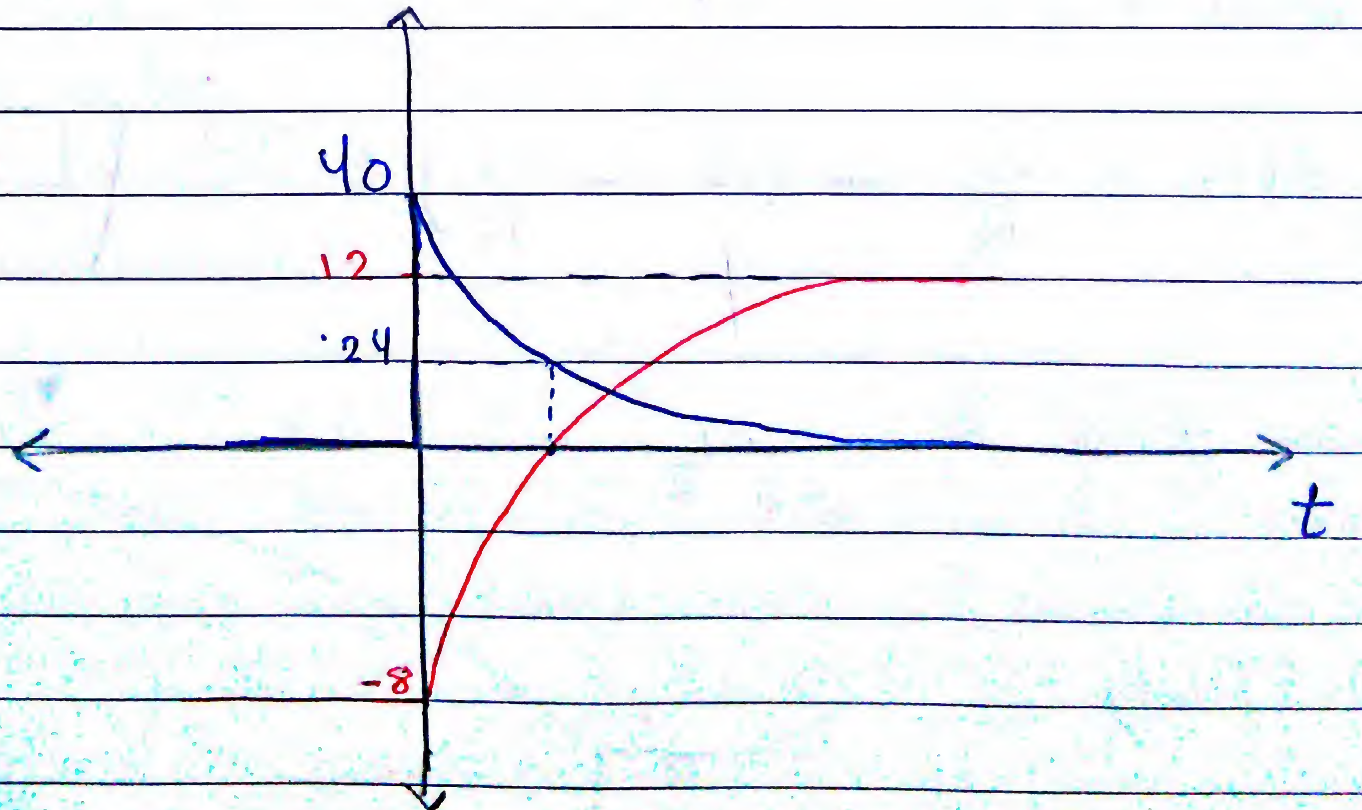
$i = 12 + (-8 - 12)\,e^{-10t}$

$i = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}$

$\boxed{i = 12 - 20\,e^{-10t}}$ $t \geq 0$

$v_L = L\,\frac{di}{dt}$

$\boxed{v(t) = 40\,e^{-10t}}$ $t > 0^+$



Date: Subject:

عند اي لحظة زمنية جهد ال الملف يساوي جهد المصدر

$\Longleftarrow$ لما التيار هيساوي الصفر

$i(0)$ at $t_1 =$

$V(t) = 24$ at t_1.

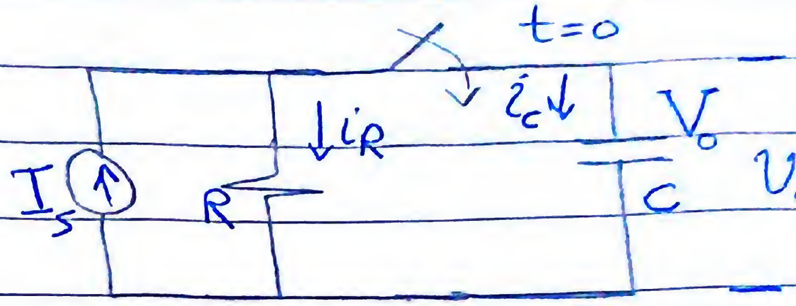
لجهد المصدر

$t_1 = 51$ ms

Subject: Lec5

Step responce of RC Circuits

$$i_C + i_R = I_S$$



$$C\frac{dV_o}{dt} + \frac{V_o}{R} = I_S$$

$(\div C)$

$$\frac{dV_o}{dt} + \frac{V_o}{RC} = I_S/C$$

$$V_o = \underbrace{I_S R}_{\text{Final (ss)}} + (\underbrace{V_o}_{IC} - \underbrace{I_S R}_{\text{Final}})e^{-t/\tau}$$

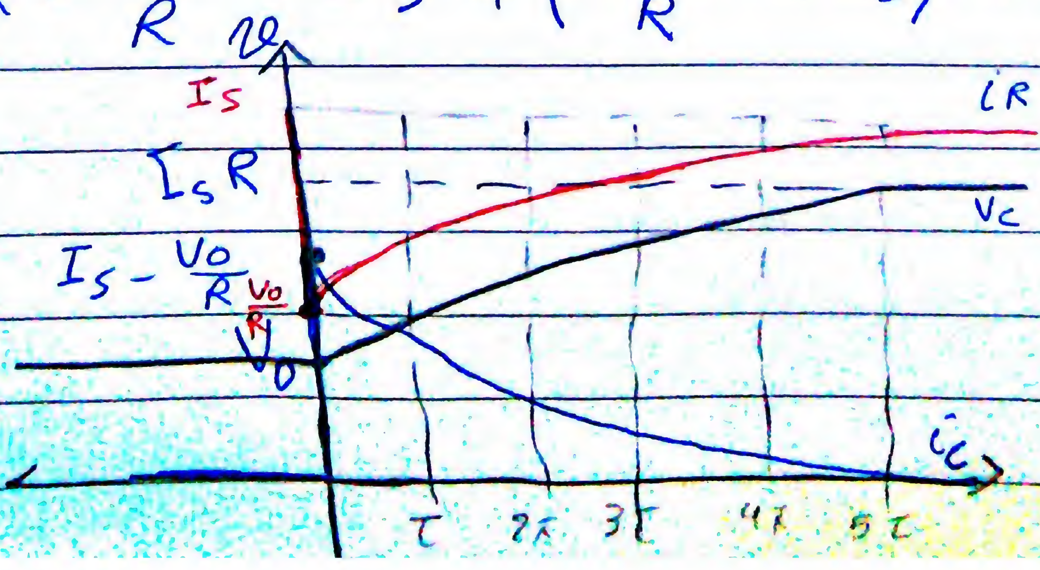
$\boxed{t \geq 0}$

initial Voltage ع المكثف → 0, +ve, -ve

$$\tau = RC$$

$$i_C = C\frac{dV}{dt} = \left(I_S - \frac{V_o}{R}\right)e^{-t/\tau} \qquad \boxed{t > 0^+}$$

$$i_R = \frac{V}{R} = I_S + \left(\frac{V_o}{R} - I_S\right)e^{-t/\tau} \qquad \boxed{t > 0^+}$$



لما توصلها بعد زمن
5 اضعاف τ

Date:

Subject:

← في ال ∞ التيار يوصل الى صفر

← تيار المقاومة كان I_s ولحظة فتح المفتاح قل
وهايرجع يزيد تاني

$$v = i_R R$$

$$V = (I_s - i_c)R$$

$$= \underbrace{I_s R}_{\text{ثابت}} - i_c R$$

$$\frac{dv}{dt} = -R\frac{di_c}{dt} \qquad (*C)$$

$$C\frac{dV}{dt} = -CR\frac{di_c}{dt}$$

$$i_C = -CR\frac{di_c}{dt}$$

$$\frac{di_c}{dt} + \frac{1}{RC}i_c = 0$$

$$i_C = K e^{-t/\tau}$$

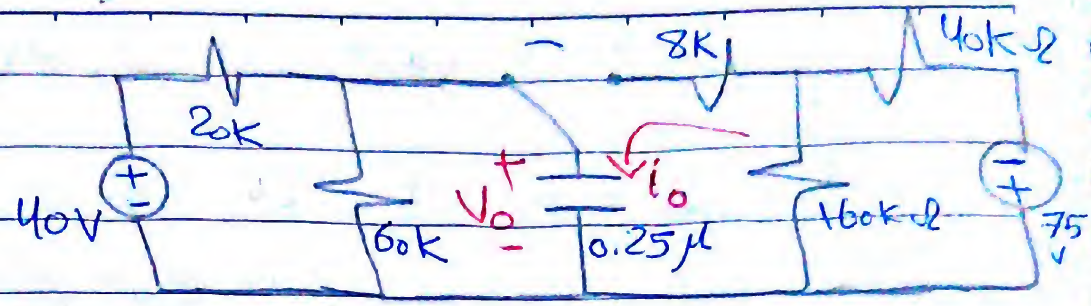
$$v = \frac{1}{C}\int i_C\, dt + V_0$$

$$U = i_R R$$

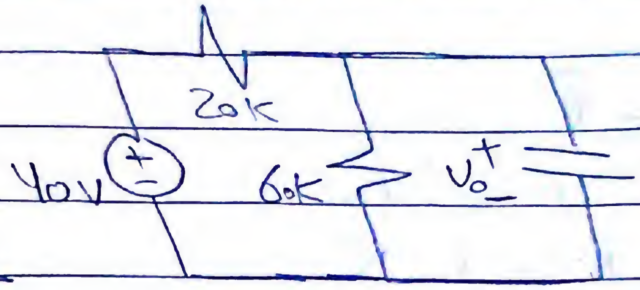
$$= (I_s - i_C)R$$

Date: Subject:

EX:

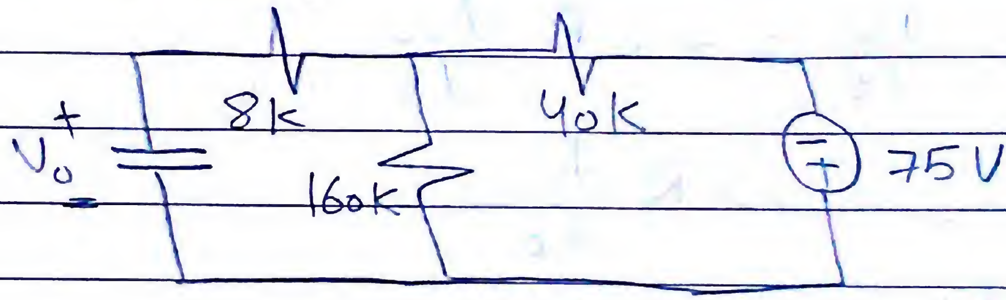


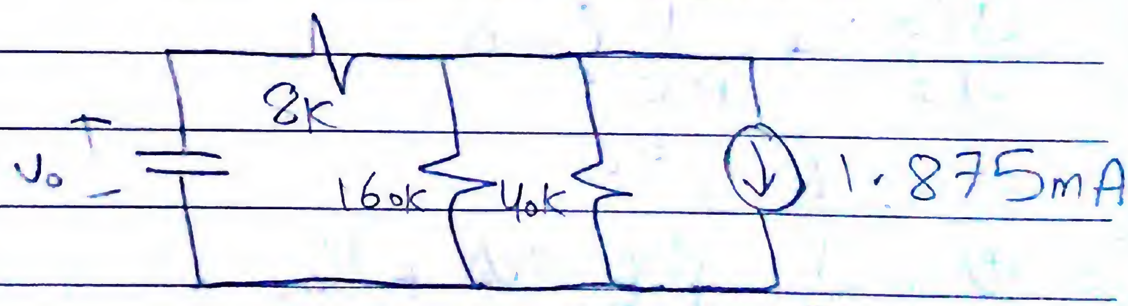
$t<0$

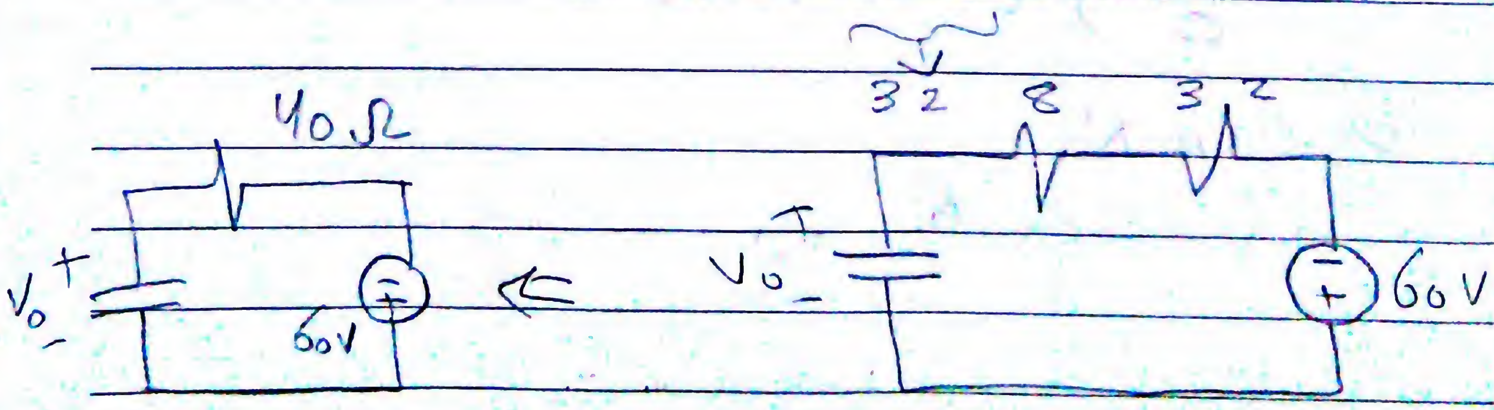


$V_o = 40 * \frac{60}{80}$

$V_o = 30$ Volt

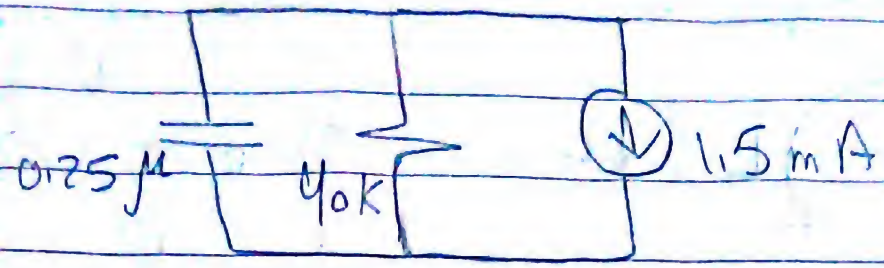






الحيا رهان لـ البالوطي

Date: Subject:

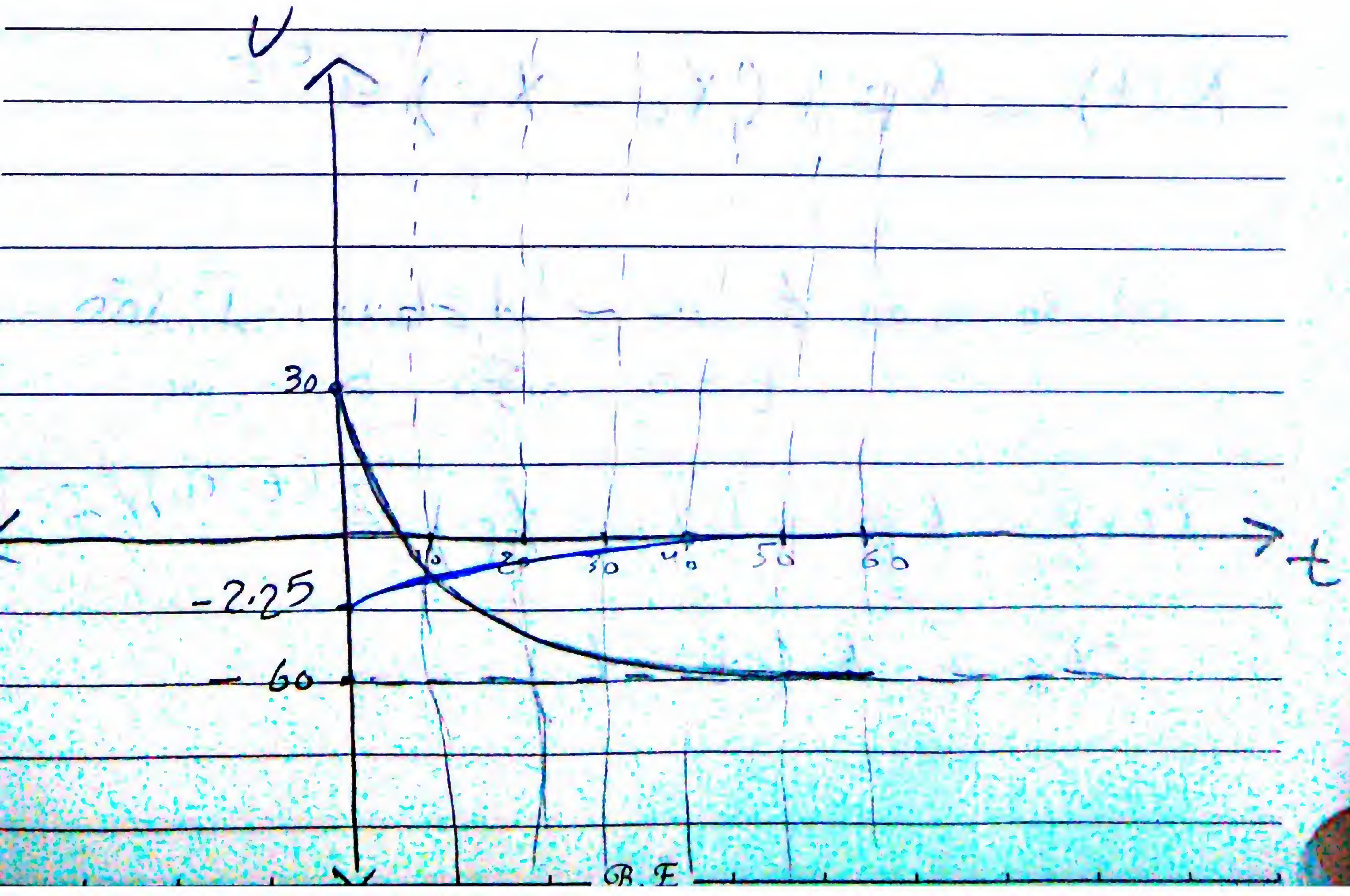


$$\tau = 0.25 \times 40 \times 10^{-6} \times 10^{3} = 0.01\ s$$

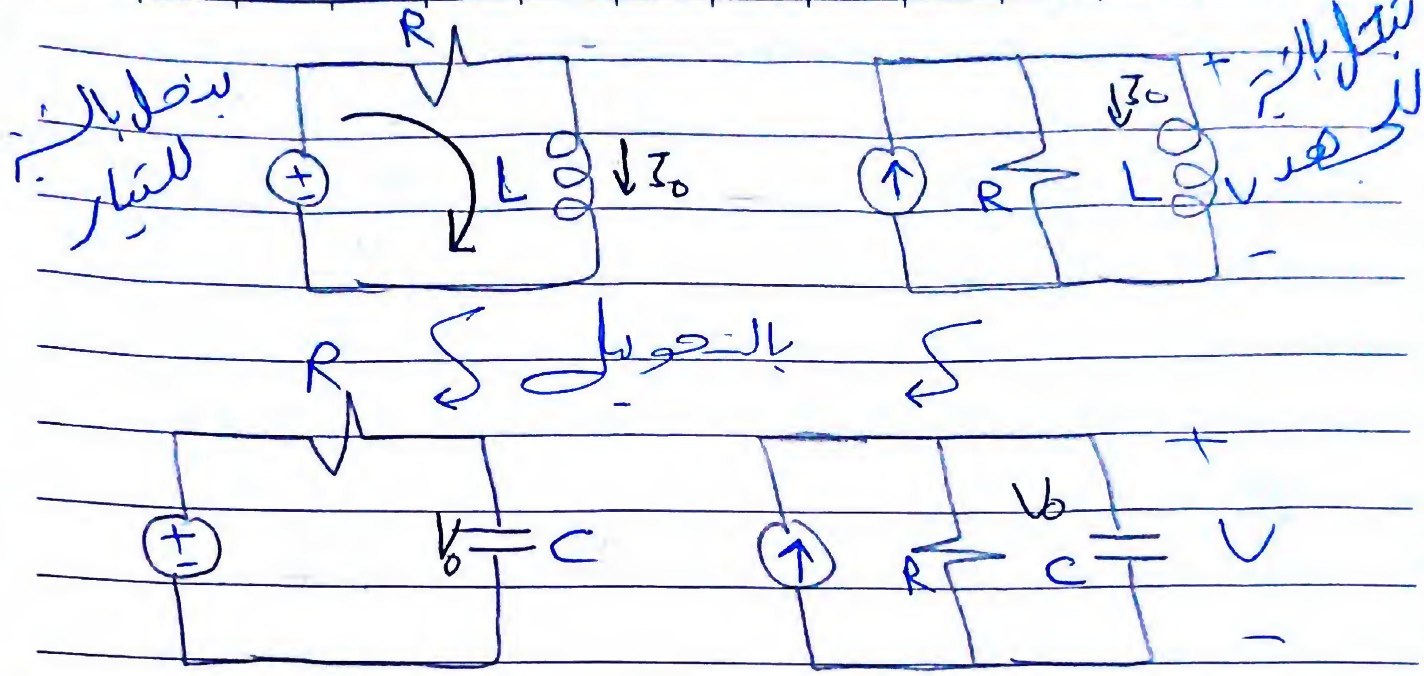
$$V_f = I_s R$$
$$= -1.5\ m \times 40\ K = -60$$

$$V_0 = I_s R + (V_0 - I_s R)\, e^{-t/\tau}$$
$$V_0 = -60 + 90\, e^{-100t}$$

$$i_0 = -2.25\, e^{-100t}\ mA$$



Date: Subject:



General Form

قد تكون جهد او تيار

$$\frac{dx}{dt}+\frac{1}{\tau}x = K \rightarrow 0 \ \& \ \text{step}$$

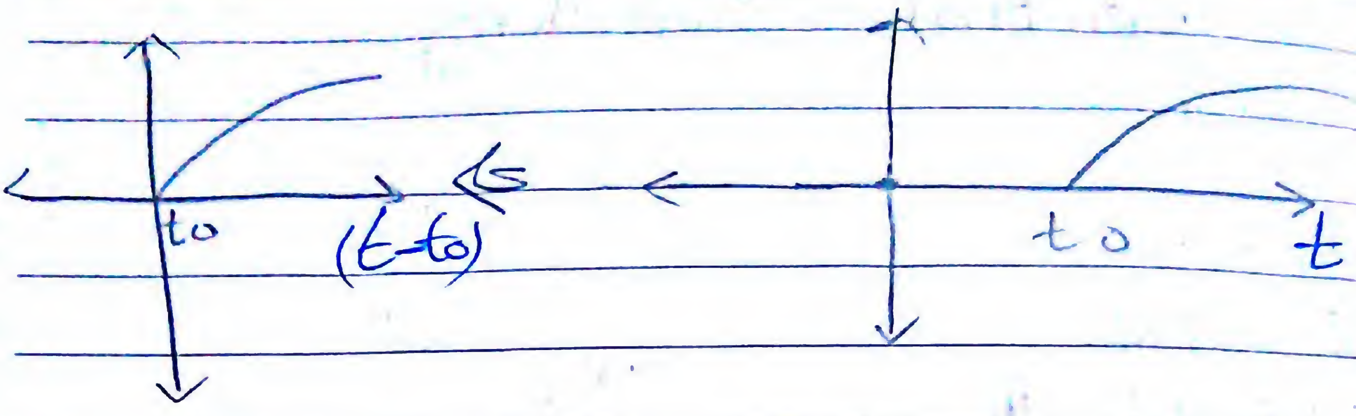
$$x(t) = x_F + (x_0 - x_F)e^{-t/\tau}$$

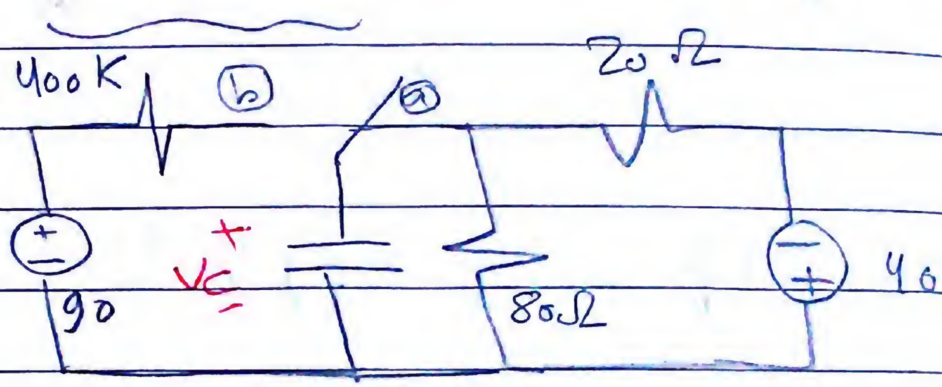
ممكن تكون نقطة البداية عند t بقيمة معينة
مش شرط تكون $t=0$.

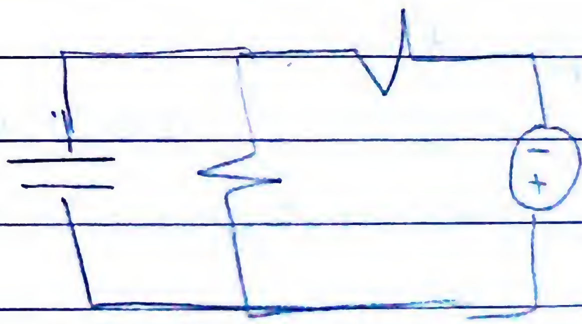
$$\boxed{x(t) = x_F + (x_0 - x_F)e^{-(t-t_0)/\tau}}$$

$$x_0 = x(t=t_0)$$

Date: Subject:

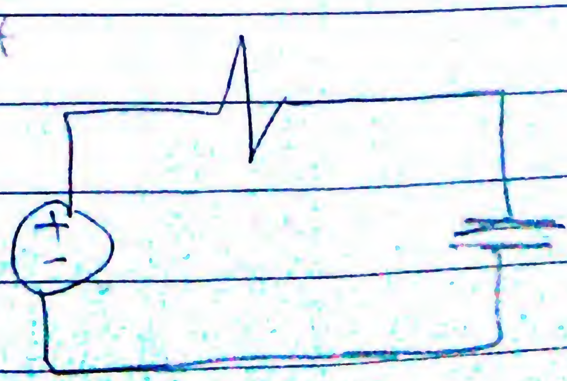






$\tau = 0.2S$

$V_C = 90 - 120e^{-5t}$



Date: Subject: Lec 6

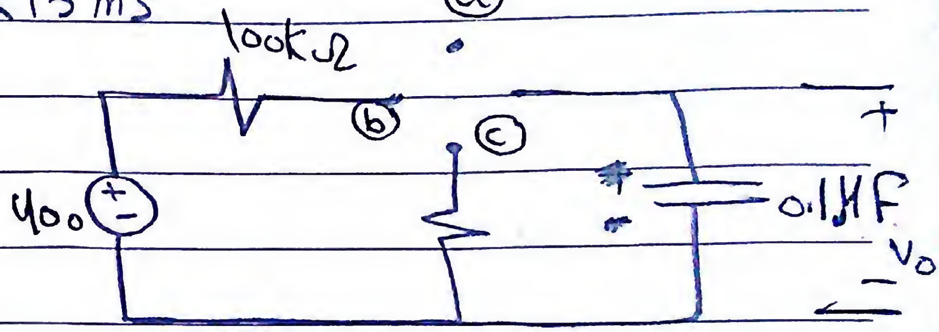
Sequential switching

تتابع

ⓐ $t < 0$

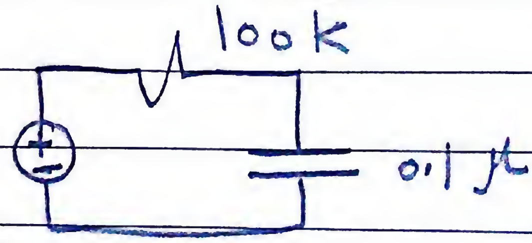
ⓑ ~~15 ms~~ $0 < t < 15\ ms$

ⓒ $15 < t < \dots$



$$V_o = V_f + (V_o - V_f)\, e^{-t/\tau}$$

① 0 — 15 ms



$V_{f_1} = 400$

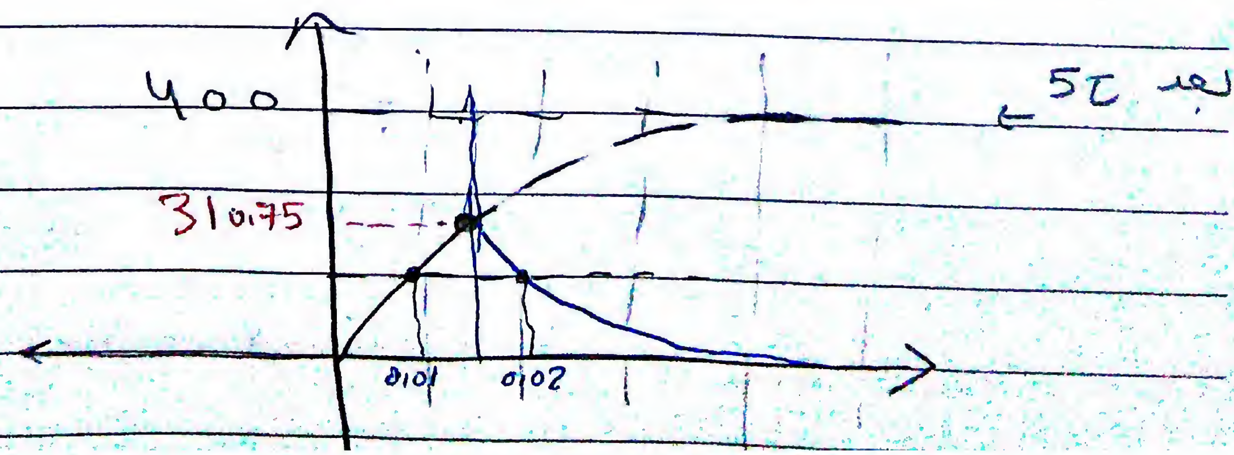
$V_{o_1} = 0 \rightarrow$ ⓐ شايفة الوضع

$\tau_1 = 10\ msec$

① $V_{o_1}(t) = 400 - 400\, e^{-100t}$

$= 400(1 - e^{-100t})$

②



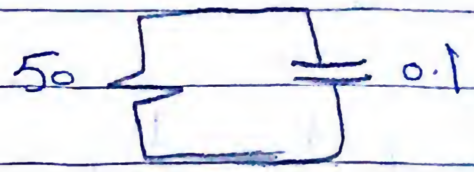
Date: Subject:

② $15ms \longrightarrow$

$V_{f2} = 0$

$V_{o2} = V_{o1}(t = 15ms)$

$= 310.75\,V$



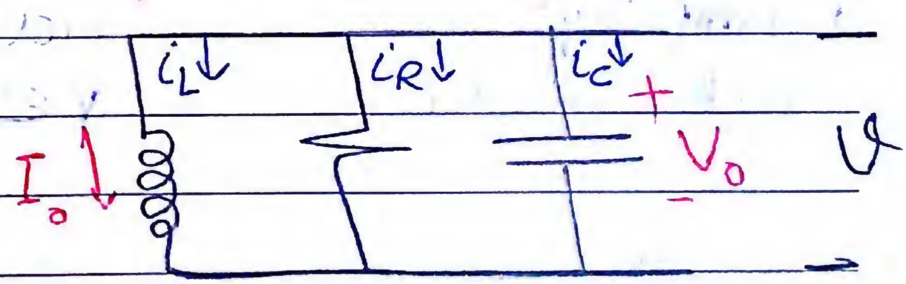
$\tau_2 = 5ms$

$$V_{o2}(t) = 310.75\, e^{-200(t - 0.015)}$$

Response of second order system

1 Parallel RLC Circuit

مجموع التيارات = صفر



$i_C + i_L + i_R = 0$

$$C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v\,dt + I_o = 0$$

معادلة فيها تفاضل وتكامل

← في حالة Step
هانجيب response
current source

← هاكتب المعادلة بدلالة الجهد المشترك بينهم

Date: Subject:

$$C\frac{d^2 v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

بقسمة الطرفين على C

$$\frac{d^2 v}{dt^2} + \underbrace{\frac{1}{RC}}_{2\alpha}\frac{dv}{dt} + \underbrace{\frac{1}{LC}}_{\omega_0^2} v = 0$$

معادلة تفاضلية خطية من الدرجة الثانية.

$$\frac{d^2 v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0$$

$$\boxed{S^2 + 2\alpha S + \omega_0^2 = 0}$$

α → Damping Ratio & Factor

ω_0 → natural Freq. resonant Freq $= \frac{1}{\sqrt{LC}}$

$$\alpha = \frac{1}{2RC}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\boxed{v = A_1 e^{S_1 t} + A_2 e^{S_2 t}}$$

A_1, A_2 ← ثوابت تكامل

Date: Subject:

$v(0)$ —— (1) $\quad V(0) = A_1 + A_2$

$\frac{dv}{dt}(0)$ —— (2) $\quad \frac{dV}{dt}(0) = S_1 A_1 + S_2 A_2$

لو ال α أقل من W_0 ← لو ال α أكبر من W_0

$\alpha^2 > W_0^2$

over damped

إخماد زائد.

responce....

$\alpha^2 < W_0^2$

under damped

responce....

$\alpha^2 = W_0^2$ → Critically damped

إخماد حرج.

~~[illegible]~~

$\alpha = \frac{1}{2RC}$ $\quad W_0 = \frac{1}{\sqrt{LC}}$ } RLC

~~[illegible]~~

Date:

Subject: Lec 7

Parallel RLC Natural

$$\alpha = \frac{1}{2RC} \text{ rad/sec} \quad \Big\{ \quad W_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$\alpha > W_0$ over damped

$\alpha = W_0$ Critical damped

$\alpha < W_0$ under damped

$$\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

A_1, A_2 يتم تصميم بـ ال initial Condition

$\rightarrow$ ① $v_{(0)} = V_0$

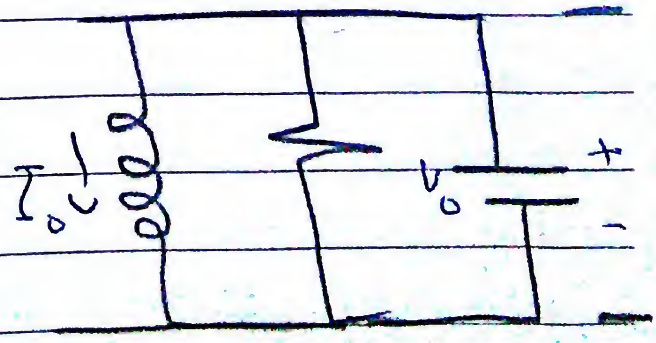
② $\frac{dv}{dt}(0)$ V/S

معدل الجهد للتغير في الزمن في زمن $t=0$

نجري عملية التفاضل اولاً ثم نعوضهم عن $t=0$

$$i_C = \frac{C\,dv}{dt}$$

$$\frac{dv}{dt} = \frac{1}{C} i_C$$



KVL:

$$i_L + i_R + i_C = 0$$

$$i_C = -(i_L + i_R)$$

$$\frac{dV}{dt} = -\frac{1}{C}(i_L + i_R) \rightarrow \text{general}$$

$$\frac{dv}{dt}(0) = -\frac{1}{C}\left[i_L(0) + i_R(0)\right]$$

$$= -\frac{1}{C}\left[I_0 + \frac{V_0}{R}\right]$$

$$V(0) = V_0 = A_1 + A_2$$

$$\frac{dv}{dt}(0) = -\frac{1}{C}\left[I_0 + \frac{V_0}{R}\right]$$

1 Over damped Case

$$\boxed{\alpha > \omega_0}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

مهم ⟸ S_2 , S_1 هايبقوا رقم سالبة ..

→ negative real

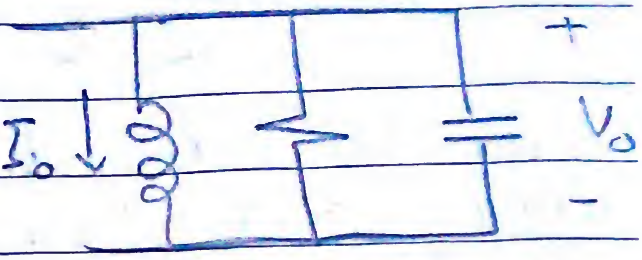
$$V(t) = A_1 e^{\overset{-ve}{S_1} t} + A_2 e^{\overset{-ve}{S_2} t}$$

EX. 8.2

$L = 50\,mH$ — مهم يقولي انه over damped
$C = 0.2\,\mu F$ — ونعرف الـ damped ratio
$R = 200\,\Omega$ — وبالتالي الـ R سهل احسبها

$V_0 = 12$ Volt

$I_0 = 30\,mA$



① احسب α و ω_0

$$\alpha = \frac{1}{2RC} = 12500 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10000 \text{ rad/s}$$

$\because \alpha > \omega_0 \therefore \rightarrow$ over damped

② نحسب S_1 و S_2

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -12500 + \sqrt{(12500)^2 - (10000)^2}$$
$$= -5000$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -12500 - \sqrt{(12500)^2 - (10000)^2}$$
$$= -20000$$

③

$$V(t) = A_1 e^{-5000t} + A_2 e^{-20000t}$$

$$\frac{dV}{dt} = -5000 A_1 e^{-5000t} - 20000 A_2 e^{-20000t}$$

Date: Subject:

$$\frac{dv}{dt}(0) = -5000A_1 - 20000A_2$$

$$V(0) = A_1 + A_2 = 12 \text{ Volt}$$

$$\frac{dv}{dt}(0) = -\frac{1}{C}\left[I_0 + \frac{V_0}{R}\right]$$

$$= -\frac{1}{0.2\mu F}\left[30mA + \frac{12V}{200}\right]$$

$$= -450000 \text{ Volt/sec}$$

$$A_1 + A_2 = 12 \rightarrow (1)$$

$$-5A_1 - 20A_2 = -450 \rightarrow (2)$$

نضرب الاولى 5 ونجمع

$$5A_1 + 5A_2 = 60$$

$$-5A_1 - 20A_2 = -450$$

$$-15A_2 = -390$$

$$A_2 = 26$$

$$A_1 = 12 - 26 = -14$$

$$\therefore V(t) = -14e^{-5000t} + 26e^{-20000t}$$

$$i_R(t) = \frac{V(t)}{R} = -0.07e^{-5000t} + 0.13e^{-20000t} \text{ A}$$

$$i_R(t) = -70e^{-5000t} + 130e^{-20000t} \text{ mA}$$

Date:

Subject:

$$i_C(t) = C\frac{dv}{dt}$$

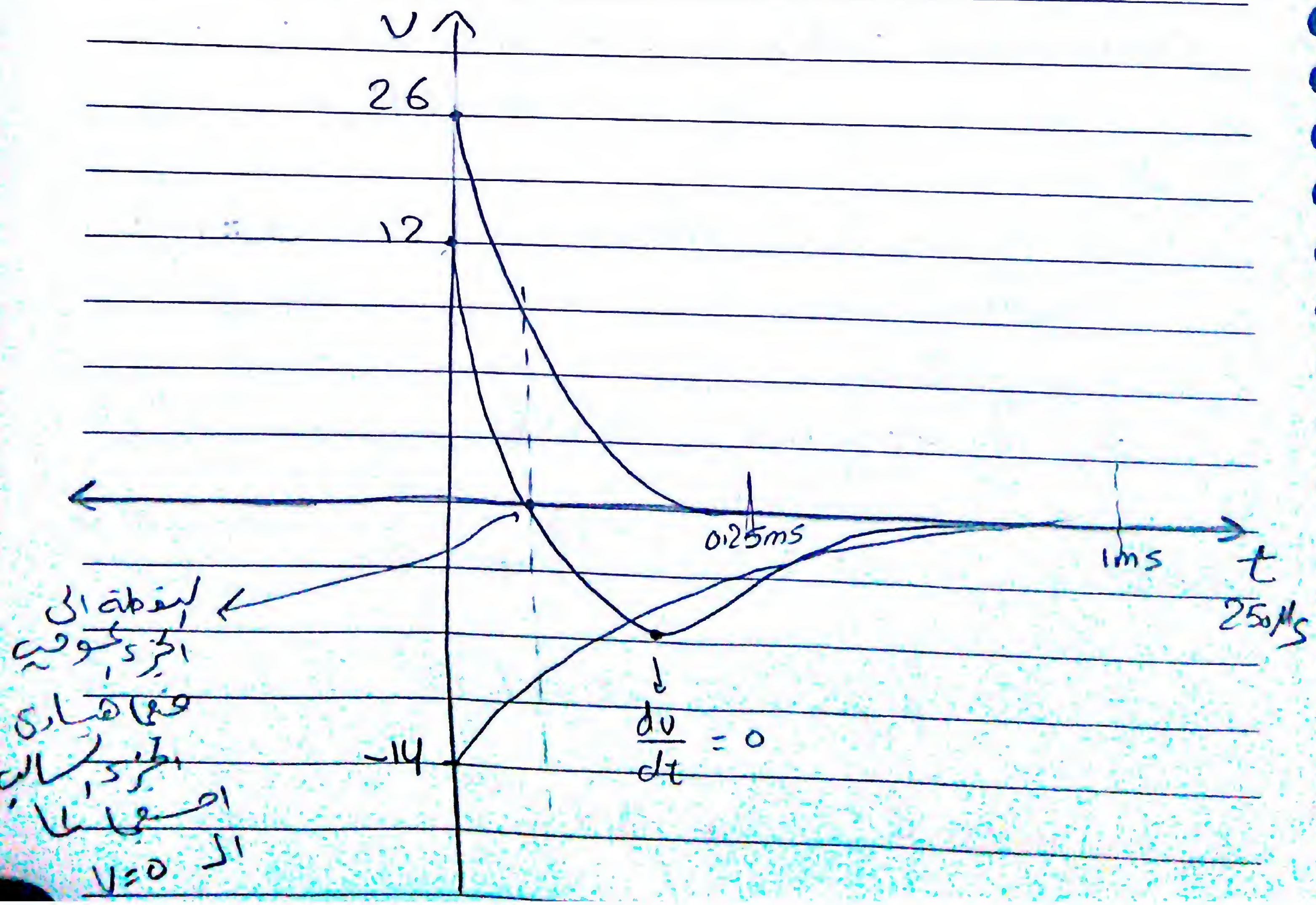
$$= 0.2\times10^{-6}\left[(-14)(-5000)\,e^{-5000t} + (26)*(20000)\,e^{-20000t}\right]$$

$$= 0.014\,e^{-5000t} - 0.104\,e^{-20000t}\ A$$

$$= 14\,e^{-5000t} - 104\,e^{-20000t}\ mA$$

$$i_L = -i_R - i_C = 56\,e^{-5000t} - 26\,e^{-20000t}\ mA$$

$$i_L = \frac{1}{L}\int v(t)\,dt + I_0$$



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الرسمه للشكل الأول $\leftarrow$ $V(t)$

لشكل $\tau = \frac{1}{5000}$ فبيكون ابطأ في الاضمحلال

وبينتهي عند $5\tau = \frac{5}{5000} = \frac{1}{1000}$

$= 1 \times 10^{-3}$ sec $= 1$ msec

الشكل الثاني $5\tau = \frac{5}{20000}$

2.5×10^{-4} sec $= 0.25$ msec

يعني في حد هاينتهي ف 1 msec والتاني هاينتهي في 0.25 mes

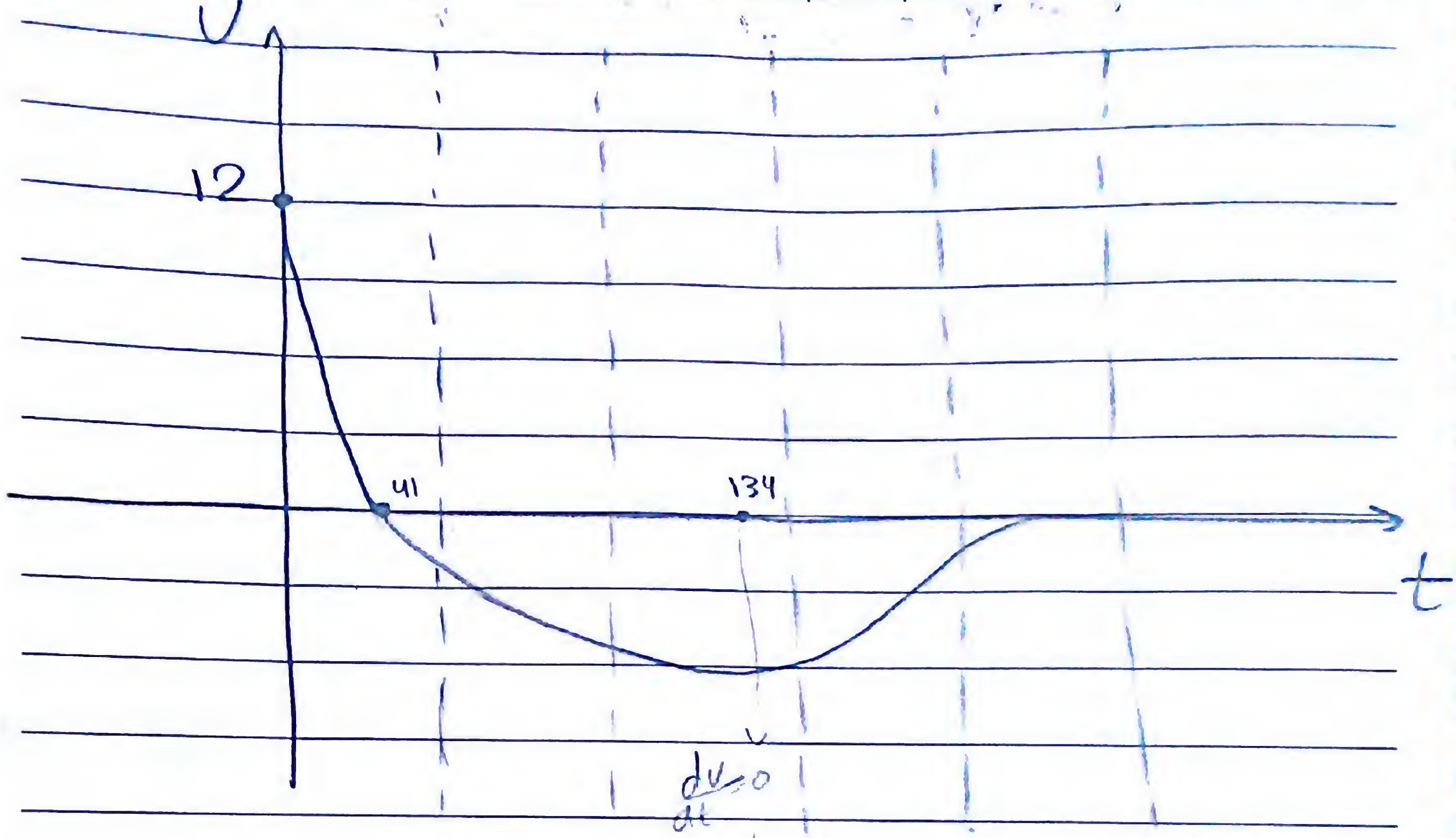
المطلوب: الرسم 250 ms

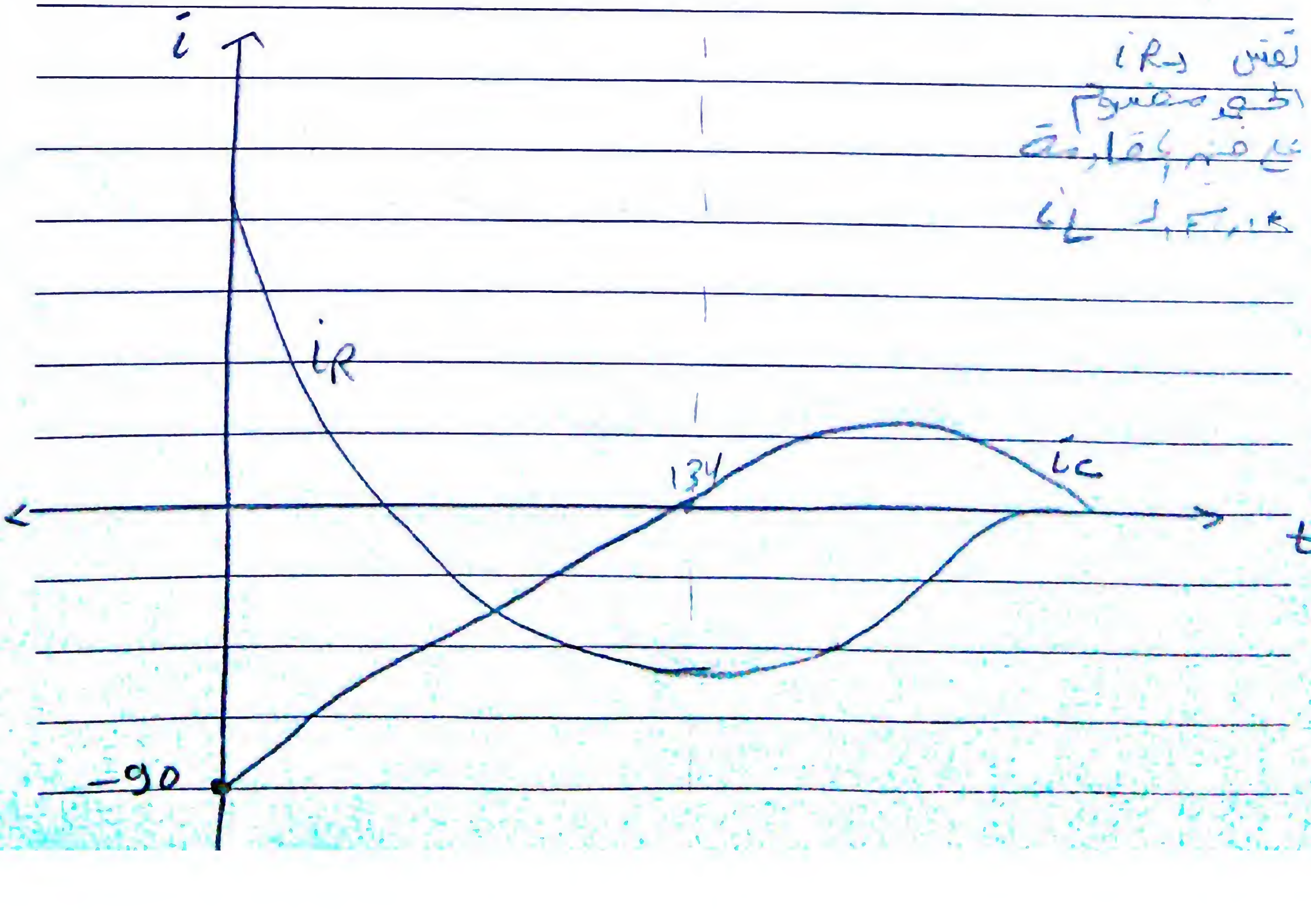
at $V = 0$

$V_{(t)} = 0$ at 41.27 MS

Date:

Subject:





نفس iR
الخط مقلوب
ع قيم مقاربة
iL لـ F.I.K

Date: Subject:

2 Under-Damped Case

$$\boxed{\alpha < w_0}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2}$$

$\alpha^2 - w_0^2 < 0 \xrightarrow{\text{اطلع}} j\sqrt{w_0^2 - \alpha^2}$

$$\therefore \; S_1 = -\alpha + j\sqrt{w_0^2 - \alpha^2}$$

$$S_2 = -\alpha - j\sqrt{w_0^2 - \alpha^2}$$

الجزء الـ real دايماً -ve

$\sqrt{w_0^2 - \alpha^2} \Longrightarrow w_d$

w_d: damped Frequency
w_0: undamped natural Frequency

Date: Subject:

$$U(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Complex . ارقام مركبة ⟵ s_1 , s_2

$$U(t) = A_1 e^{(-\alpha + J w_d) t} + A_2 e^{(-\alpha - J w_d) t}$$

$$= A_1 e^{-\alpha t} e^{+J w_d t} + A_2 e^{-\alpha t} e^{-J w_d t}$$

$$= e^{-\alpha t} [A_1 e^{+J W_d t} + A_2 e^{-J W_d t}]$$

$$e^{\pm j\theta} = \cos\theta \pm j \sin\theta$$ Eular

$$U(t) = e^{-\alpha t} [A_1 \cos\theta + j A_1 \sin\theta + A_2 \cos\theta - j A_2 \sin\theta]$$

$$\theta = W_d t$$

$$\therefore U(t) = e^{-\alpha t} [A_1 \cos W_d t + j A_1 \sin W_d t + A_2 \cos W_d t - J A_2 \sin W_d t]$$

$$= e^{-\alpha t} [(A_1 \cos W_d t + A_2 \cos W_d t) + j (A_1 \sin W_d t - A_2 \sin W_d t)]$$

Date: Subject:

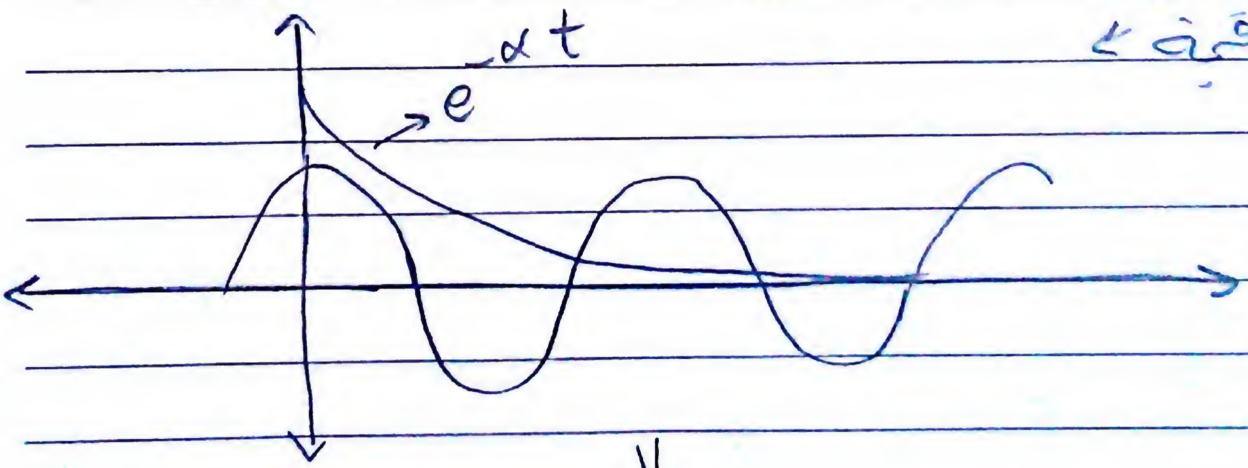
$$\therefore\ V(t) = e^{-\alpha t}\left[\underbrace{(A_1+A_2)}_{\text{real}}\cos w_d t + \underbrace{j(A_1-A_2)}_{\text{real}}\sin w_d t\right]$$

$$\boxed{V(t) = e^{-\alpha t}\left[B_1 \cos w_d t + B_2 \sin w_d t\right]}$$

$$\frac{dV}{dt}(0) = -\alpha B_1 + w_d B_2$$

$$V(0) = B_1$$

لو B_2 ، B_1 ليهم قيمة ؟



$\Downarrow$